19.14

"what is an analogue of the continuous-field conservation law $\nabla^a(T_{ab}\kappa^b) = 0$, for a discrete system of particles where 4-momentum is conserved in collisions? "

Notice κ^b is a vector field. Not a constant vector. Not that the idea of a constant vector field means anything in curved spacetime, anyway - since parallel transport of a vector depends on the path it moves along.

So, try $p_a\kappa^a$, p_a being the energy-momentum form for a particle.

There are 10 linearly independent Killing vector fields in Minkowski space - same dimension as the Poincare' group. (the Killing vector fields are infinitesimal metric-preserving transformations on the space)

For the translational vector fields $\kappa^a = \{\delta_1^a, \delta_3^a, \delta_3^a, \delta_4^a\}, p_a\kappa^a = \{E, p_x, p_y, p_z\}$, and those are conserved.

I'm assuming, at least for now, the particles don't have forces on them.

There are 3 rotational fields, $\kappa^a = (0, y, -x, 0)$ which is a rotation around the z-axis, and similar rotations around x and y axes. These give you conservation of angular momentum.

There are also 3 Killing vector fields from the Lorentz transform

$t' = \beta(t - vx)$,

$x' = \beta(x - vt)$, where $\beta = 1/\sqrt{1 - v^2}$

You get the Killing vector - the infinitesimal Lorentz transform - by differentiating wrt v:

$\partial t'/\partial v|_{v=0} = -x$,

$\partial x'/\partial v|_{v=0} = -t$.

So $\kappa^a = (-x, -t, 0, 0)$ is a Killing vector field. $p_a\kappa^a = -Ex + tp_x$, and that's conserved if there are no forces.

So $p_a\kappa^a$ is conserved for one particle. When there's a collision, the energy-momentum forms are contracted with the same vector before and after so the total dot product is conserved through the collision. You can see how this might provide the underpinning for the stress-energy tensor.

If there are forces on the particle, the forces would have their own stress-energy tensor and from that perhaps you could get something that added to the energy-momentum vectors of the particles over their trajectories in spacetime would be constant over the trajectory of the particle, when dotted with the Killing vector field.

Laura