Q 19.18: from the Bianchi identity
\[ \nabla_a R_{bcde}^e + \nabla_b R_{cade}^e + \nabla_c R_{abde}^e = 0 \]
derive
\[ \nabla^a (2R_{ab} - Rg_{ab}) = 0 \]

A: Rewrite the Bianchi identity as
\[ \nabla_a R_{bcde} g^{ef} + \nabla_b R_{cade} g^{ef} + \nabla_c R_{adeb} g^{ef} = 0 \]
Contract f with a, and it turns into
\[ \nabla_e R_{edcb} + \nabla_b R_{deca} g^{ea} + \nabla_c R_{deab} g^{ea} = 0 \]
raising b,
\[ \nabla^e R_{eabc}^b + \nabla^b R_{decb}^e + \nabla_c R_{de}^e b = 0 \]
contract d with b, so
\[ \nabla^e R_{eabc}^b + \nabla^b R_{deca}^e - \nabla_c R_{eb}^e b = 0 \]
renaming some indices and writing it in terms of the Ricci tensor,
\[ 2\nabla^a R_{ab} - \nabla_b R_c^c = 0, \text{so} \]
\[ \nabla^a (2R_{ab} - Rg_{ab}) = 0! \]

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