

The Hamiltonian  $H$  at equilibrium is a function of the  $p_i$ 's and  $q_j$ 's, where  $q_j$  is a generalized coordinate and  $p_i = \partial L / \partial \dot{q}_i$  is the generalized momentum.

And he says, in the Taylor expansion at equilibrium, there aren't second order terms that are a  $p_i q_j$ . That is the only part of the exercise that was puzzling to me, because "in theory" you could have  $H = p^2 - pq + q^2$ , which has a minimum at  $p = q = 0$  and is positive for all other  $(p, q)$ .

What I realized though is that if you get the Hamiltonian from the Lagrangian, terms that look like  $\dot{q}_j f(q_i)$  are eliminated, because  $L = \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - H$ . And at equilibrium,  $\dot{q}_i = 0$ . So that's why you don't have quadratic terms in the Taylor expansion of  $L$  at equilibrium that look like  $\dot{q}_i q_j$ .

In his example  $p_i = \dot{q}_i$ , although in general it wouldn't be, for example for the Hamiltonian for a nonrelativistic particle in an electromagnetic field.

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