

The Hamiltonian H at equilibrium is a function of the p_i 's and q_j 's, where q_j is a generalized coordinate and $p_i = \partial L / \partial \dot{q}_i$ is the generalized momentum.

And he says, in the Taylor expansion at equilibrium, there aren't second order terms that are a $p_i q_j$. That is the only part of the exercise that was puzzling to me, because "in theory" you could have $H = p^2 - pq + q^2$, which has a minimum at $p = q = 0$ and is positive for all other (p, q) .

What I realized though is that if you get the Hamiltonian from the Lagrangian, terms that look like $\dot{q}_j f(q_i)$ are eliminated, because $L = \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - H$. And at equilibrium, $\dot{q}_i = 0$. So that's why you don't have quadratic terms in the Taylor expansion of L at equilibrium that look like $\dot{q}_i q_j$.

In his example $p_i = \dot{q}_i$, although in general it wouldn't be, for example for the Hamiltonian for a nonrelativistic particle in an electromagnetic field.

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