

Roger Penrose seems to conserve parity, in the sense that he is oblivious to + and - signs.

Exercise 22.9 is about the Heisenberg picture of quantum mechanics, where states don't change in time, the operators change instead. So $Q|\psi(t)\rangle$ becomes $U_t^{-1}QU_t|\psi\rangle$, where U_t is the time evolution operator. The operator Q becomes $Q_H = U_t^{-1}QU_t$.

Exercise 22.9 asks you to show that $i\hbar\frac{dQ_H}{dt} = HQ_H - Q_HH$. Well here's how you show the opposite.

$$i\hbar\frac{dQ_H}{dt} = i\hbar\frac{dU_t^{-1}}{dt}QU_t + i\hbar U_t^{-1}Q\frac{dU_t}{dt}$$

$i\hbar\frac{dU_t^{-1}}{dt} = -HU_t^{-1}$ since U_t^{-1} is evolution in the $-t$ direction.

Similarly $i\hbar\frac{dU_t}{dt} = HU_t$.

So $i\hbar\frac{dQ_H}{dt} = -HU_t^{-1}QU_t + U_t^{-1}QHU_t$.

Differentiating $U_t^{-1}U_t = I$, you get $-HU_t^{-1}U_t + U_t^{-1}HU_t = 0$, so $H = U_t^{-1}HU_t$, or $U_tH = HU_t$.

So $i\hbar\frac{dQ_H}{dt} = -HU_t^{-1}QU_t + U_t^{-1}QU_tH = Q_HH - HQ_H$.

Laura