In Exercise 24.3 he says "for positive A there are many solutions of $-\nabla^2\psi = A\psi$, which tail off suitably towards ∞ so that the norm $||\psi||$ remains finite."

I was puzzled by this because I thought he was saying it would be a free-space solution (where the potential is 0 everywhere) for the Schrödinger equation. A free-space solution would also be an energy eigenstate, a stationary state. And it isn't possible to have an energy eigenstate in free space that's normalizable.

But he doesn't mean free-space solutions. I thought of a particle bouncing around inside an ∞ hard spherical shell. The magnitude of its momentum vector doesn't change, just the direction changes. So, a particle in a spherical container, with potential 0 inside and ∞ outside, is an example of what he's talking about. To solve the Schrödinger equation for this potential, you find a spherical Bessel function that has a 0 at the boundary of the sphere, and you match it with a 0 wavefunction outside the sphere. This gives you a stationary solution, and then you can find the energy of the solution from the Schrödinger equation.

Such a solution has definite $p_x^2 + p_y^2 + p_z^2$, because $p_x^2 + p_y^2 + p_z^2$ is proportional to ∇^2. $-\nabla^2\psi = A\psi$ (for some A) inside the sphere, since the potential is 0 inside, and outside the sphere also $-\nabla^2\psi = A\psi$ since $\psi = 0$.

This solution isn't really physical, because of the ∞ potential. I suppose the derivatives of ψ aren't continuous at the boundary of the sphere, otherwise these spherical Bessel functions would be possible stationary states for a free particle. But having definite $p_x^2 + p_y^2 + p_z^2$ is unphysical anyway, so this is of no concern.

Actually a solution for a particle inside an ∞ hard shell of whatever shape should be an eigenstate of ∇^2.

Laura