Relativistic invariance of the Dirac equation:

He gives the Dirac operator as $\gamma^a \partial/\partial x_a$. That's a typo, it should be $\gamma^a \partial/\partial x^a$. The coordinates x^a have the index on the top.

Since the coordinates x^a are contravariant, the partial derivative operators $\partial/\partial x^a$ are covariant, that means they transform the opposite way from x^a. So $\gamma^a \partial/\partial x^a$ is Lorentz invariant, since it's contracted over a contravariant and a covariant index. The rest mass on the righthand side is also Lorentz invariant.

If the Dirac equation is Lorentz transformed it becomes

$$\acute{\gamma}^a \partial \acute{\psi}/\partial \acute{x}^a = -iM\acute{\psi}$$

$\acute{\gamma}^a = L^a_b \gamma^b$ where L^a_b is a Lorentz transform. Because the algebra of the γs is defined by $\gamma_a \gamma_b + \gamma_b \gamma_a = -2g_{ab}I$, and g_{ab} is Lorentz invariant, the $\acute{\gamma}^a s$ have the same commutators as the $\gamma^a s$, (so the new algebra is isomorphic)

The wavefunction ψ is a scalar so it's invariant under a Lorentz transform, so that would mean the whole equation is relativistically invariant.

Actually he explains later that ψ is really a spinor in the Dirac equation. So what happens to a spinor under Lorentz transform? Not explained yet (though Wikipedia said).

Laura