

Exercise 27.16 asks you to show why a connected 3-space can't be isotropic about 2 distinct points without being homogeneous.

Counterexample, though. Suppose the space is S^3 , the 3-dimensional sphere - say, sitting in 4-space as $x^2 + y^2 + z^2 + w^2 = 1$.

Then let the 2 separate points be antipodal points on the sphere. For example $x = (1, 0, 0, 0)$ and $-x = (-1, 0, 0, 0)$.

You could have a matter distribution that was isotropic around both of these points, because a rotation around x is also a rotation around $-x$! But it doesn't have to be homogeneous. The matter density could be a function of distance from x or $-x$.

A hard exercise, to uncreate the sphere!

I wonder how this exercise might be modified to be true. What you want (to prove it) is to be able to nudge one point a little by rotating around the other point a little.

I wonder if there are 3-manifolds with a rotation about a point that has more than 2 fixed points.