27.9 How long does it take for all the gas molecules in a 1-liter box to spontaneously go back into a little box that's 1/10 of a liter volume?

A: You need *some* time scale to start with. I figured using the time scale on which the gas molecule positions are randomized is reasonable. Say 1 second, for gas molecule positions in a 1-liter box.

So in 1 second, the probability they'll all spontaneously go back into the little box is about $10^{-10^{22}}$, since there are about 10^{22} gas molecules in a liter.

Say that $P(t)$ is the probability that after a time t, they haven't all gone back into the little box yet, and p is the probability per unit time that they'll all decide to bop back into the box. $P(t + \Delta t) = (1 - p\Delta t)P(t)$ so $dP/dt = -pP(t)$, $P(t) = e^{-pt}$. When $pt = 1$, $P(t) = 1/e$, they've probably all gone back into the box.

If $t = 10^{10^{22}}$sec, $P(t)$ is $finally > 1/2$! You see why you don't need to know the speed the molecules are moving! You want to make an ridiculously too small estimate for how long it'd take the molecules to get randomized? How about a nanosecond $= 10^{-9}$ second? Light goes about 3×10^{8}m/sec, so it would go across the 1-liter box in about 1/3 nanoseconds. We know the gas molecules aren't going relativistically fast, but suppose they are and they randomize in a nanosecond.

Then it would take about $t = 10^{10^{22}-9}$sec for all the gas to go back into the box! It "hardly makes a difference" even though it's a billionth of the original estimate, because the time's so huge it's more like the log of the time that's relevant.

If you divided a 1-liter box in half, there would be *some* variation in the numbers of molecules in each half. There wouldn't be exactly 5×10^{21} molecules on each side. The normal fluctuation, the standard deviation, is about the square root of 5×10^{21}. So there'd be a pressure difference of about 1 in 20 billion.

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