Q: Find a formula for gravitational time dilation near a body mass M, given a timelike Killing vector κ.

A: Suppose you have a particle of mass m falling from rest at ∞, straight towards M. It's following a geodesic so the product $p_a\kappa^a$ is conserved (see exercise 19.14). At ∞, $\kappa = (1, 0, 0, 0)$ and $p_a = (mc^2, 0, 0, 0)$. At a distance R from M, the particle has kinetic energy mMG/R. So its energy is now $m(c^2 + MG/R)$. Since the spacetime is stationary, the space components of κ are 0, $\kappa = (\kappa^0, 0, 0, 0)$. By conservation of $p_a\kappa^a, mc^2 = \kappa^0(m(c^2+MG/R))$, so $\kappa^0 = 1/(1+MG/Rc^2)$.

So why is this time dilation? It is in this sense at least: $p_0 = \hbar\omega$ for a light ray, where ω is frequency. Coming out of the gravitational field, by the conservation of $p_a\kappa^a$, the frequency ω decreases by a factor of $1 + MG/Rc^2$. That means that time seems to be going more slowly at the source of the light.

Also, κ is a vector field that describes how a spacelike (constant-time) slice of the spacetime evolves in time. You would evolve the spacelike slice an infinitesimal amount in time, by applying the κ vector to move each point a little along a geodesic in the time direction. Since the κ vector is shorter closer to M, that means that time moves more slowly closer to M.