Another, easier way of showing that $\int \rho_1(\vec{x})\phi_2(\vec{x})dx^3 = \int \rho_2(\vec{x})\phi_1(\vec{x})dx^3$.

Integrating by parts, $-\frac{1}{4\pi}\int \rho_1(\vec{x})\phi_2(\vec{x})dx^3 = \int \nabla^2\phi_1(\vec{x})\phi_2(\vec{x})dx^3 = -\int \nabla\phi_1(\vec{x}) \cdot \nabla\phi_2(\vec{x})dx^3$, assuming that $\nabla\phi_1(\vec{x})\phi_2(\vec{x}) \to 0$ at ∞.

$\int \nabla\phi_1(\vec{x}) \cdot \nabla\phi_2(\vec{x})dx^3$ is symmetrical in ϕ_1 and ϕ_2, so that proves it.