Exercise 30.13 asks why there aren't any energy eigenstates for a "free" particle, i.e. no potential energy term in the Schrödinger equation.

Because the hypothetical wavefunction is an energy eigenstate, you can just look at the time-independent Schrödinger equation, which is

$$\nabla^2\psi = \frac{2mE}{-\hbar^2}\psi$$

So $p_x^2 + p_y^2 + p_z^2$ has a definite value of $2mE$.

That means that ψ isn't normalizable. To see that, try normalizing it via a Fourier transform. $\psi(\vec{x}) = \int \Psi(\vec{p})e^{ixp}dp$ (up to a constant multiple) so

$$\int \psi(\vec{x})\overline{\psi(\vec{x})}dx = \int \Psi(\vec{p})e^{i\vec{x}\cdot\vec{p}}\overline{\Psi(\vec{q})}e^{-i\vec{x}\cdot\vec{q}}dpdqdx$$

If you integrate over x first you get a constant multiple of a delta function from the integral $\int e^{i\vec{x}\cdot(\vec{p}-\vec{q})}dx$. Dropping the constant multiple, since all we want to show is that the integral is infinite, we get

$$\int \Psi(\vec{p})e^{i\vec{x}\cdot\vec{p}}\overline{\Psi(\vec{q})}e^{-i\vec{x}\cdot\vec{q}}dpdqdx = \int |\Psi(\vec{p})|^2 dp$$

But $p_x^2 + p_y^2 + p_z^2 = 0$ except on the surface of a 2-sphere. So the density $\Psi(\vec{p})$ is a product of at least one δ function, and $\int |\Psi(\vec{p})|^2 dp$ is an integral over the square of a delta function.

The integral of the square of a delta function is infinite, as you can see by trying to normalize the 1-dimensional momentum eigenstate $\phi(x) = e^{ixp} = \int \delta(p-q)e^{ixq}dq$. Then $\int \phi(x)\overline{\phi(x)}dx$ is the integral of a square of a delta function $\int |\delta(p-q)|^2 dq$ (up to a constant multiple), which is infinite since the norm of e^{ixp} is infinite.