Q: Try to explain why a small linear perturbation in the Schwarzschild metric would become infinite as it approaches O.

A: I'm not sure if I got this one right, but this is my best effort:

In Minkowski space the y-coordinate is imaginary: So $t = iy$ is real. In the Euclideanization, the y-coordinate is real and we're assuming that the perturbation is an analytic function of $z = x + iy$.

$ln(x + t) = ln(z) = ln(\sqrt{x^2 + y^2}) + i\theta = ln(\sqrt{x^2 - t^2}) + \tau/\beta$ so $x + t = e^{\tau/\beta}\sqrt{x^2 - t^2}$, where β is the black hole's Hawking temperature.

Since the perturbation is analytic in the Euclideanization, it can be expressed as a Taylor series in $z = x + t$. Since $z^n = e^{n\tau/\beta}(x^2 - t^2)^{n/2}$, a Taylor series component $a_n z^n$ increases exponentially with τ, at a constant distance $s = \sqrt{x^2 - t^2}$ from O. I think the components $a_n z^n$ are what he means by "eigenmodes of $\partial/\partial\tau$". So the perturbation does $\rightarrow \infty$ within a constant distance from O.

However if you think of a line $x + t = c$, which approaches O in a Euclidean sense as $c \rightarrow 0$, $z^n = c^n$ on that line, so a $a_n z^n$ component of the perturbation doesn't diverge as you approach the origin in a Euclidean sense.

I looked at an article on perturbations in the Schwarzschild metric, in Physical Review June 24 1966, by PC Peters. It didn't mention any problem like this. He derived a formula for the perturbation to the metric caused by a single small mass on a timelike world-line. It was in the original Schwarzschild metric, not the simplified conformally-equivalent version used in the book, but it didn't apparently diverge at the event horizon. But he didn't make the assumption that the perturbation was analytic in the Euclideanized space, of course.

So I don't know if I've missed something, or there's a mistake in the exercise, or a Penrosian vagueness that has caused me to fill in the blanks wrong. Or maybe this is what he meant and I'm only wrong about thinking I might be wrong! If that makes sense :)

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