Q: Show how a manifold with a metric g_{ab} that respects the complex structure J_b^a is a symplectic manifold with symplectic form S_{ab} and $dS_{ab} = 0$.

A: I think what he means by the metric preserving the complex structure is, first, that if you "multiply two vectors v^a and w^b by i", i.e. by the J matrix, the dot product $v \cdot w$ is not affected, i.e. $v^a g_{ab} w^b = v^a J_a^c g_{cd} J_b^d w^b$.

Since this is true for general v and w, $g_{ab} = J_a^c g_{cd} J_b^d$. Multiplying each side by J_f^a, $J_f^a g_{ab} = (J_f^a J_a^c) g_{cd} J_b^d = -\delta_f^c g_{cd} J_b^d = -g_{fd} J_b^d$.

So if you define $S_{fb} = J_f^a g_{ab}$, then $S_{fb} = -S_{bf}$, and S is the symplectic form that was asked for.

"Preserving the complex structure" also means that the Levi-Civita connection ∇ derived from g_{ab} is "complex-linear", that is $\nabla_a (J_b^c v^b) = J_b^c \nabla_a v^b$.

Since $\nabla_a (J_b^c v^b) = \nabla_a (J_b^c) v^b + J_b^c \nabla_a v^b$ by the Leibnitz rule, that means that $\nabla_a (J_b^c) v^b = 0$. Since this is true for all v, $\nabla_a J_b^c = 0$.

Since $\nabla_a g_{bc} = 0$, also $\nabla_a S_{bc} = \nabla_a (J_b^d g_{dc}) = 0$.

As he says in Sec. 14.6, exercise 14.23, the exterior derivative $\mathbf{d}$ is independent of the choice of connection. So that shows right away that $\mathbf{d} S_{ab} = 0$.

$\nabla_a J_b^c = 0$ also means that parallel transport preserves the complex structure. If $t^a \nabla_a v^b = 0$, which means that v^b is parallel transported along the curve t^a, then also $t^a \nabla_a (J_c^b v^c) = 0$, so that "$iv^b$ " is also parallel transported.