This one is very cute and simple. Suppose F_{ab} is self-dual and G_{ab} is anti-self-dual, so ${}^*F_{ab} = iF_{ab}$ and ${}^*G_{ab} = -iG_{ab}$.

${}^*F^{ab} = \frac{1}{2}\epsilon^{abpq}F_{pq}$, so

$$F^{ab} = \frac{-i}{2}\epsilon^{abpq}F_{pq}.$$

So

$$F^{ab}G_{ab} = \frac{-i}{2}\epsilon^{abpq}F_{pq}G_{ab}.$$

Similarly, ${}^*G^{ab} = \frac{1}{2}\epsilon^{abpq}G_{pq}$, so

$$G^{ab} = \frac{i}{2}\epsilon^{abpq}G_{pq}.$$

So

$$G^{ab}F_{ab} = \frac{i}{2}\epsilon^{abpq}G_{pq}F_{ab}.$$

$G^{ab}F_{ab} = G_{ab}F^{ab}$, since you can always raise one contracted index, lowering the other at the same time.

So

$$\frac{-i}{2}\epsilon^{abpq}F_{pq}G_{ab} = \frac{i}{2}\epsilon^{abpq}G_{pq}F_{ab}.$$

But if you interchange p, q and a, b, you see this means that $G^{ab}F_{ab} = 0$.

In the same way, you get 0 if you contract a self-dual F_{fg} with an anti self-dual Weyl tensor C_{abcd}, on any pair of indices. There are two extra indices on the Weyl tensor; they just go along for the ride.

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