33.19 Why is the twistor commutator $Z^\alpha\overline{Z}_\alpha - \overline{Z}_\alpha Z^\alpha$ real?

$\langle\psi|(Z^\alpha\overline{Z}_\alpha - \overline{Z}_\alpha Z^\alpha)\phi\rangle = \langle(Z^\alpha\overline{Z}_\alpha - \overline{Z}_\alpha Z^\alpha)\psi|\phi\rangle$, since when you move operators from the right to the left side of an amplitude, you take the complex conjugate and reverse the order of operation. So $Z^\alpha\overline{Z}_\alpha - \overline{Z}_\alpha Z^\alpha$ is the dual of itself.

So if $Z^\alpha\overline{Z}_\alpha - \overline{Z}_\alpha Z^\alpha = \beta I$, where β is a complex number, then
$\overline{\beta}\langle\psi|\phi\rangle = \langle\beta\psi|\phi\rangle = \langle(Z^\alpha\overline{Z}_\alpha - \overline{Z}_\alpha Z^\alpha)\psi|\phi\rangle = \langle\psi|(Z^\alpha\overline{Z}_\alpha - \overline{Z}_\alpha Z^\alpha)\phi\rangle = \langle\psi|\beta\phi\rangle = \beta\langle\psi|\phi\rangle$,
so β must be real.

This is only a guess ...

Laura