33.19 Why is the twistor commutator $Z^a\overline{Z}_a - \overline{Z}_aZ^a$ real?

Since when you move operators from the right to the left side of an amplitude, you take the complex conjugate and reverse the order of operation. So $Z^a\overline{Z}_a - \overline{Z}_aZ^a$ is the dual of itself.

So if $Z^a\overline{Z}_a - \overline{Z}_aZ^a = \beta I$, where $\beta$ is a complex number, then

$$\overline{\beta}\langle\psi|\phi\rangle = \langle\beta\psi|\phi\rangle = \langle(Z^a\overline{Z}_a - \overline{Z}_aZ^a)\psi|\phi\rangle = \langle\psi|(Z^a\overline{Z}_a - \overline{Z}_aZ^a)\phi\rangle = \langle\psi|\beta\phi\rangle = \beta\langle\psi|\phi\rangle,$$

so $\beta$ must be real.

This is only a guess ...

Laura