I'm reading Lars Hormander's book on multivariable complex analysis and I'm puzzled by something.

In Theorem 2.3.2' the suppositions are:

Let Ω be a bounded open set in $\mathbf{C}^n$, $n > 1$, such that the complement of $\bar{\Omega}$ is connected and $\partial\Omega \in C^4$. Let ρ be a real valued function in C^4 such that $\rho = 0$ precisely on $\partial\Omega$ and $\mathrm{grad}(\rho) \neq 0$ on $\partial\Omega$. Let $u \in C^4(\bar{\Omega})$ and $\bar{\partial}u \wedge \bar{\partial}\rho = 0$ on $\partial\Omega$. Then, one can find an analytic function $U \in C^1(\bar{\Omega})$ such that $U = u$ on $\partial\Omega$.

I don't know what he means by $\partial\Omega \in C^4$, but that isn't the question. I figure maybe if it's a typo maybe it'll become clear what it means later.

Otherwise: $\bar{\Omega}$ is the closure of Ω

$\partial\Omega$ is the boundary of Ω

$C^k(S)$ is k times continuously differentiable complex-valued functions on S.

$$\bar{\partial}u = \sum_{i=1}^{n} \frac{\partial u}{\partial \bar{z}_i} d\bar{z}_i$$

Now here's the question. He says in the proof that by assumption, $\bar{\partial}u = h_0\bar{\partial}\rho + \rho h_1$, $h_0 \in C^3(\bar{\Omega})$.

You could do a least squares fit of $\bar{\partial}u$ to a multiple of $\bar{\partial}\rho$. But that has a problem: the least squares fit would blow up when $\mathrm{grad}(\rho) = 0$, which it would be somewhere in Ω.

So how do you determine h_0 in Ω? $K = \{z \in \bar{\Omega} : \mathrm{grad}(\rho) = 0\}$ is compact. There is a smooth function S with $S(z) = 1$ for $z \in K$, and $\mathrm{support}(z) \subset \Omega$. Define $f(z) = S(z)\bar{\partial}\rho(z) + (1 - S(z))\bar{\partial}u(z)$, and define h_0 as the least squares fit of $f(z)$ to $h_0(z)\bar{\partial}\rho(z)$. Then $h_0(z)$ doesn't diverge when $\mathrm{grad}(\rho) = 0$.