

Paint drying

He says that the vector field $\frac{\partial f}{\partial \omega^0} \frac{\partial}{\partial \omega^1} - \frac{\partial f}{\partial \omega^1} \frac{\partial}{\partial \omega^0}$ operates on $\mathbb{P}\mathbb{T}$.

In the usual vector notation this is $(-\frac{\partial f}{\partial \omega^1}, \frac{\partial f}{\partial \omega^0}, 0, 0)$. This works as a vector field on $\mathbb{P}\mathbb{T}$ because the homogeneity degree of $\frac{\partial f}{\partial \omega^0} \frac{\partial g}{\partial \omega^1} - \frac{\partial f}{\partial \omega^1} \frac{\partial g}{\partial \omega^0}$ is the same as the homogeneity degree of g . Which is good, because a function on $\mathbb{P}\mathbb{T}$ has to be of homogeneity degree 0 to be well-defined, and $\frac{\partial f}{\partial \omega^0} \frac{\partial g}{\partial \omega^1} - \frac{\partial f}{\partial \omega^1} \frac{\partial g}{\partial \omega^0}$ is a function on $\mathbb{P}\mathbb{T}$.

While the paint is drying, ω is shifted to ω' , which satisfies the differential equation $\frac{d\omega'(s)}{ds} = \left(-\frac{\partial f(\omega')}{\partial \omega^1}, \frac{\partial f(\omega')}{\partial \omega^0} \right)$, and $\omega'|_{s=0} = \omega$. The value of ω when the paint is dry is ω' at some value of s , say $s = 1$.

He says that an area measure can be defined on the ω -fibers, which are twistors (ω, π) with a fixed value of π . That's because shifting the ω -coordinates doesn't change the magnitude of $\mathbf{d}\omega^0 \wedge \mathbf{d}\omega^1$. Shifting an infinitesimal ds , $\mathbf{d}\omega^0 \wedge \mathbf{d}\omega^1$ changes to

$\left(-\frac{\partial^2 f}{\partial \omega^0 \partial \omega^1} ds + 1, \frac{\partial^2 f}{\partial (\omega^0)^2} ds \right) d\omega^0 \wedge \left(-\frac{\partial^2 f}{\partial (\omega^1)^2} ds, \frac{\partial^2 f}{\partial \omega^1 \partial \omega^0} ds + 1 \right) d\omega^1$, and the terms that are linear in ds cancel.