I wondered, reading section 33.9, what first analytic sheaf cohomology is in the complex plane. It turns out that all open sets in C have vanishing first analytic sheaf cohomology.

The idea of the first analytic sheaf cohomology group is that you have an open cover U_α of the set, and holomorphic functions $f_{\alpha\beta}$ defined on the intersections $U_\alpha \cap U_\beta$, such that on triple intersections $f_{\alpha\beta} + f_{\beta\gamma} + f_{\gamma\alpha} = 0$ (i.e. the coboundary of the set of $f_{\alpha\beta}$'s is 0), and a set of $f_{\alpha\beta}$'s is considered equivalent to 0 iff $f_{\alpha\beta} = g_\alpha - g_\beta$, with g_α analytic on U_α and g_β analytic on U_β.

If the open set is C, and the open cover is $U_1 = z : \text{Re}(z) < \pi$ and $U_2 = z : \text{Re}(z) > \pi/2$, then $e^{1/sinz}$ is holomorphic on $U_1 \cap U_2$. The vanishing first analytic sheaf cohomology on C means you could express $e^{1/sin(z)}$ as the sum of two functions, one analytic in U_1, one analytic in U_2. It's pretty surprising! I looked up the proof in Lars Hormänder's book "An introduction to complex analysis in several variables".

What Roger Penrose says about trivial cohomology meaning that the contour integral is 0 for a function that's a coboundary, evidently doesn't apply to the complex plane.

Laura